

Integrated Global Optimization Model for Electrical Cables in Offshore Wind Farms

Journal:	<i>IEEE Transactions on Sustainable Energy</i>
Manuscript ID	TSTE-00826-2019
Manuscript Type:	Transactions
Date Submitted by the Author:	26-Jul-2019
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Technical Topic Area :	Wind turbine generators < Transactions on Sustainable Energy, System optimization
Key Words:	Wind power generation, Cables, Optimization methods, MILP, Export cables, Collection systems, Offshore

Integrated Global Optimization Model for Electrical Cables in Offshore Wind Farms

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Abstract—A MILP program for integrated global optimization of electrical cables systems in Offshore Wind Farms (OWFs) is presented. Electrical cables encompass the cable layout in collection systems to interconnect Wind Turbines (WTs), and transmission systems to couple Offshore Substations (OSSs) to the Onshore Connection Point (OCP). The program is solved through a modern branch-and-cut solver, demonstrating the ability to tackle large-scale instances (hundreds of WT and several OSSs). The model supports as objective function initial investment plus economic losses due to total electrical power losses. The importance and functionality of incorporating electrical losses is demonstrated, along with the need to optimize simultaneously the cable layout, OSSs location, and transmission cables. The method is tested for three case studies. Results show that: (i) points near the global optimum, with an imposed maximum tolerance, are calculable in reasonable computational time, and (ii) the integrated model is more efficient than a benchmark approach consisting on enumeration.

Index Terms—Offshore wind energy, Cable layout, Global optimization, Mixed integer linear programming, Medium voltage cables, High voltage cables.

I. INTRODUCTION

COST reductions and operational performance enhancement are the main current challenges for designing, and running Offshore Wind Farms (OWFs). Europe (mainly), Asia, and recently North America (first large-scale installations expected in 2023 [1]) have contributed to the rapid proliferation of this technology for electrical power generation. The share of OWFs worldwide, with respect to the total installed power of wind energy (591 GW), is prognosed to increase from the actual 4% to more than 10% in 2025, reaching a global installed capacity of of 100 GW. Electrical power cables represent a crucial component of the Balance of Plants (BoP) in OWFs, representing roughly 10% of the total capital expenses [2]. Between 2018 and 2028 more than 19,000 km of cables for collection systems are projected to be installed, with an estimated worth of £5.36bn [3]. Likewise, required export cables with lengths in the order of hundred of kilometers are the trend given the distancing of Offshore Substations (OSSs) to the Onshore Connection Point (OCP). Similarly, economies of scale pushes the development of large-scale OWFs, having more than 100 Wind Turbines (WTs), while increasing their rated power.

The importance of electrical cables in OWFs radicate not

only in the evident cost weight, but also in the operational performance of the whole project. Points of single failure due to electro-thermal and mechanical stress, and their mitigation, is one of the most studied topics in the field nowadays. Therefore, special attention must be addressed to designing systems involving electrical cables, such as the collection system, and the export system, while simultaneously accounting for the interrelation among them.

The full integrated design of electrical cables in OWFs comprise the following aspects: (i) limited capacity of the cables' thermal rating, (ii) topological design of the cable layout, (iii) capital expense optimization in both the collection and the transmission systems, (iv) diminution of total electrical power losses, (v) clustering of WT to OSSs, and (vi) geographical location of the OSSs. As demonstrated in [4], an instance of the main problem involving only the points (i), (ii), and (iii) falls into the category of NP hard problems [5], mapping to a modified version of a Capacitated Minimum Spanning Tree (C-MST). By deduction, the main problem dealing simultaneously with the six aspects, inherit the complexity with additional design variables, turning this task into a very challenging subject in the context of computational optimization in OWFs.

In order to tackle the electrical cables design in OWFs, available methods can be classified as follows [6]. (i) Heuristics methods are all those algorithms that are bounded in computation time while developing a solution by construction in sequential steps [7]. (ii) Metaheuristics can be defined as improvement heuristics, which find fast initial feasible points (ideally), and by means of iterative processes, intend to improve them using different stochastic operators [8], [9]. (iii) Global optimization offers the enormous advantage to provide mathematical certifications for optimality in case of convex problems. However, for combinatorial problems, the required computational time is unknown and unpredictable, and for the range of modern large-scale OWFs, memory capacities start becoming an issue as well [10]. (iv) Combination of matheuristics, providing frameworks to cope with these challenges by limiting smartly the search space, in combination with mathematical programs providing feasible points or the global optimum [4].

To the best of the authors' knowledge, only the following works have tackled the full integrated design of electrical cables for OWFs using global optimization: MILP model with Benders decomposition [11], MILP model [12], MILP model with progressive contingency incorporation [13], and MIQP model [14]. Nevertheless, the focus of these works has not been directed towards the application for large-scale

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Manuscript received XXX, XXX; revised XXX, XXX.

OWFs (more than 100 WTs and several OSSs), mainly due to the NP-Hardness nature of the problem. Other works such as [15] (with Particle Swarm Optimization), [16] (with Genetic Algorithm), propose metaheuristic models for providing feasible points, albeit given the intrinsic features of the used methods, there is no possibility to infer about the quality of the obtained solutions.

In virtue of the above, we propose an efficient MILP model embedded in an algorithmic framework, able to compute global optimum solution points (or near to it) in reasonable computational time, for the full integrated design of electrical cables in large-scale OWFs. Additionally, the model quickly finds feasible points. Likewise, the proposed model supports a combined objective function defined by total system costs, i.e. initial investment and total electrical power losses of both the collection and transmission systems simultaneously.

The paper is structured as follows: In the Section II, the main optimization problem is formulated including objective function, and constraints definition. Later on in the Section III, the modelling aspects are explained in details, followed by the whole framework description in Section IV, where the full method is compacted into a single main iterative algorithm. Computational experiments are performed in Section V and the work is finalized with the conclusions in Section VI.

II. OPTIMIZATION MODEL

We develop a model that supports any number of WTs, α , and any number of OSSs, β . Define the set $\mathcal{N}_o = \{\mathcal{N}_{o1}, \dots, \mathcal{N}_{o\beta}\}$, where an element \mathcal{N}_{oi} is, in turn, an index set representing candidate locations for the i th OSS with $i \leq \beta$. In this way, $\beta_t = \sum_{1 \leq i \leq \beta} |\mathcal{N}_{oi}|$ defines the total number of potential locations to place OSSs (denoted $\mathcal{N}_{\beta_t} = \{1, \dots, \beta_t\}$), given a required number β , and individual candidate set, \mathcal{N}_{oi} , for each $i \in \mathcal{N}_\beta = \{1, \dots, \beta\}$.

Likewise, let $\mathcal{N}_w = \{\beta_t + 1, \dots, \beta_t + \alpha\}$ denote the index set representing the WTs. Hence, the whole set of points (OSSs candidate locations and WTs locations) is given as, $\mathcal{N} = \{0\} \cup \mathcal{N}_{\beta_t} \cup \mathcal{N}_w$, where the node 0 represents the OCP. The aim is to obtain the optimum design of the cable layout for a full OWF, modelled as β spanning-trees, connecting the WTs between each other, and towards the desired number of OSSs, along with the transmission cables from OSSs to the OCP. Furthermore, as OWFs cover a relatively large area, it is common that they include forbidden zones - defined as spaces where no WTs or cables can be placed - which must be considered as well. The geographical location of the WTs is fixed, calculated in the micro-siting optimization process in order to maximize the energy produced in the OWF [17]. The distance between two points i and j , is defined as d_{ij} .

The complete directed graph $G(\mathcal{N}, \mathcal{A}, \mathcal{D})$ comprises the input sets, where \mathcal{N} represents the vertex set, \mathcal{A} the set of available arcs arranged as a pair-set, and \mathcal{D} the set of associated weights for each element $a \in \mathcal{A}$.

The different types of cables being considered are stored in \mathcal{T}_c , and \mathcal{T}_e , for the collection and export systems, respectively. The set \mathcal{T}_c relates to the attribute sets \mathcal{U}_c , and \mathcal{C}_c , representing the

capacity (in terms of maximum number of supported WTs), and total capital expenditures per unit of length, respectively. In general, the greater the cable capacity, the greater the capital cost.

The first set of variables in the model are binary: z_{il} x_{ij} , and y_{ij}^k . Choosing the candidate location l for the OSS i is done through the variable z_{il} , which is equal to one if selected, and zero otherwise. If the arc connecting i , and j is active ($i \in \mathcal{N} \setminus \{0\} \wedge j \in \mathcal{N} \setminus \{0\}$), then $x_{ij} = 1$. Finally, in case $x_{ij} = 1$, the number of WTs connected is defined by y_{ij}^k , where k models the number of WTs rooted at i , including the one at j . A set of positive integer variables is required as well; σ_i , and σ_{il} represents the number of WTs connected to the OSS i , and at the corresponding candidate location l , respectively.

The number of x_{ij} , and y_{ij}^k variables scale quadratically with the number of WTs. Therefore, reduction strategies are important to limit the size of the model in terms of variables and constraints. A procedure proposed in [18] and [19] is used. The largest individual cable capacity is found as $U_c = \max \mathcal{U}_c$; consequently, the maximum attributable value of k for $i \in \mathcal{N}_{\beta_t}$ is equal to $f(i) = U_c$, in contrast, for elements $i \in \mathcal{N}_w$ is $f(i) = U_c - 1$. This is because, intuitively, a cable only can be used at maximum capacity if connected to a OSS. Finally, all redundant arcs are suppressed, along with those interconnecting OSSs, as only the common industry practice of point-to-point connections from offshore to onshore points is considered. Overall, the original graph $G(\mathcal{N}, \mathcal{A}, \mathcal{D})$ is reduced to $G_r(\mathcal{N}, \mathcal{A}_r, \mathcal{D}_r)$.

A. Cost coefficients

Designing the electrical cables system in OWFs is manifold: not only arcs must be selected while also choosing the cable type to do so, but total electrical power losses must be considered as they may impact the design. Total electrical power losses are function of the selected arc, cable type, and generated power.

Hence, the straight-forward way to include simultaneously all these aspects is to incorporate in the variable y_{ij}^k the cable type, as for instance transforming the variable to $y_{ij}^{k,t}$. Variable $y_{ij}^{k,t}$ would model whether the arc $a_r = (i, j)$ is selected or not, connecting k WTs through cable type t . Secondly, losses would have to be mathematically explicitly expressed in the objective function. This raises two issues: (i) the number of variables increase linearly with the number of cable types $|\mathcal{T}_c|$, and (ii) ohmic losses are non-linear, therefore simplifications must be assumed for expressing it in a linear model.

A body of actions to circumvent these limitations, while being able to solve the defined problem, are implemented. The approach basically decouples the arc selection and losses minimization from the cable assignment decision-making problem. These techniques are explained in the following sections for both the collection system (cable layout), and transmission system (point-to-point connection).

1) *Collection system cables:* The reduced graph $G_r(\mathcal{N}, \mathcal{A}_r, \mathcal{D}_r)$ contains all required information for the cable layout. For y_{ij}^k , the length of arc (i, j) is known, along

with the number of WTs connected; this makes possible to evaluate the whole set of available cables in polynomial running time for each of the arcs (such as $i \neq 0 \wedge j \neq 0$), to select that which minimizes the objective function. All cost coefficients are non-negative.

Let the term $(c_{c_t} \cdot d_{ij})$ represent the initial investment plus installation costs (per metric unit) of cable t , to join points i and j (cost per metric unit). The parameters $c_{c_t} \in \mathcal{C}_c$ are obtained from the exponential regression function given in [20] (excluding installation costs).

To incorporate the cost of total electrical power losses, a discounted cash flow metric is considered. The required parameters are $|\mathcal{M}|$, $l_{ij}^{\mu,t,k}$, c_p , and r , meaning the project lifetime (years), total power losses at year μ for cable t (in MWh, see Section III-C), cost of energy (€/MWh), and discount rate (p.u.), respectively.

Bearing this in mind, for each (i, j) such as $i \neq 0 \wedge j \neq 0$, the following optimization model is formulated and solved independently by enumeration [19].

$$c_{ij}^k = \min \sum_{t \in \mathcal{T}_c} x_{ij}^t \cdot \left(c_{c_t} \cdot d_{ij} + \sum_{\mu=1}^{|\mathcal{M}|} \frac{l_{ij}^{\mu,t,k} \cdot c_p}{(1+r)^\mu} \right) \quad (1)$$

$$\text{s.t.} \quad \sum_{t \in \mathcal{T}_c} x_{ij}^t = 1 \quad (2)$$

$$x_{ij}^t \cdot (S_{ij}^{k,t} - S_{r_t}) \leq 0 \quad \forall t \in \mathcal{T}_c \quad (3)$$

$$x_{ij}^t \in \{0, 1\} \quad \forall t \in \mathcal{T}_c \quad (4)$$

Note that this set of problems are always feasible as k is limited by U_c . Equation (2) ensures that exactly one cable type is selected. Equation (3) guarantees that the capacity of cable t is not violated; where $S_{ij}^{k,t}$ is the power through arc (i, j) , when k turbines are connected in j using cable t , and S_{r_t} is the rated power of t (see Section III-A, and Section III-B). Lastly, Equation (4) defines the nature of the problem's variables.

After solving the model from (1) to (4) (maximum $U_c \cdot |\mathcal{N}|^2$ times), the cost coefficients c_{ij}^k are calculated, and the corresponding cable type t is unequivocally determined as per x_{ij}^t .

2) *Transmission system cables:* Arcs (i, j) such as $i \in \{0\} \wedge j \in \mathcal{N}_{\beta_t}$ are also definable beforehand. The cost of the transmission cables is function of the total length and installed power; the former is known, but the latter is also output of the main optimization model. Likewise, in contrast to the collection system cable layout, there are no binary variables explicitly representing whether certain number of WTs are connected to a OSS or not. In this case, the variables σ_{il} are used to estimate the cost for a specific candidate location $l \in \mathcal{N}_{\beta_t}$, associated to the OSS $i \in \mathcal{N}_\beta$.

Let the variable $x_{0j}^{t,n}$ represent if arc $(0, j)$ is active using n cables t in parallel or not (a limit of $n = 3$ is a reasonable practical constraint to ensure feasibility). The methodology uses the following optimization model, where the target is to obtain a linear cost function in terms of the WTs connected

to a OSS:

$$\min \sum_{n=1}^3 \sum_{t \in \mathcal{T}_c} x_{0j}^{t,n} \cdot \left(c_{e_{t,n}} \cdot d_{0j} + \sum_{\mu=1}^{|\mathcal{M}|} \frac{l_{0j}^{\mu,t,k,n} \cdot c_e}{(1+r)^\mu} \right) \quad (5)$$

$$\text{s.t.} \quad \sum_{n=1}^3 \sum_{t \in \mathcal{T}_c} x_{0j}^{t,n} = 1 \quad (6)$$

$$\sum_{n=1}^3 x_{0j}^{t,n} \cdot (S_{0j}^{k,t,n} - S_{r_t}) \leq 0 \quad \forall t \in \mathcal{T}_c \quad (7)$$

$$x_{0j}^{t,n} \in \{0, 1\} \quad \forall t \in \mathcal{T}_c \wedge n \in \{1, 2, 3\} \quad (8)$$

The mathematical program from (5) to (8) resembles that described from (1) to (4). In fact, each equation is mirrored by the order of appearance. Nevertheless, in the case of the transmission cables, most of the parameters and variables are re-indexed accounting for n , which is the number of cables per connection. Contrary to collection system cables, in transmission level parallel cables are installed in practice. Figure 1 presents the basic algorithm for calculating the transmission cables linear cost function. The idea behind is to sequentially increase the number k of turbines connected to a specific OSS location, solving (5) to (8), and gathering all results to finally obtain the best linear function fitting the data by least squares method.

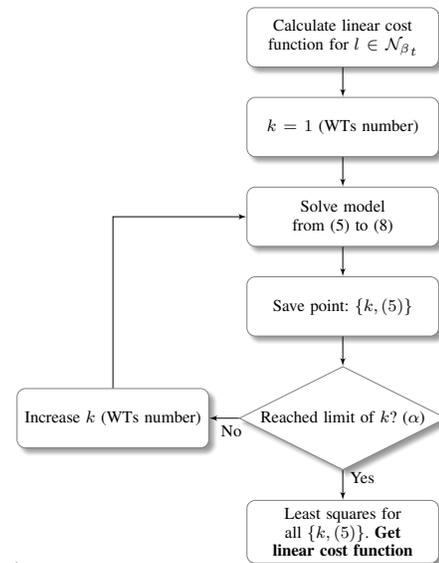


Fig. 1: Flowchart for calculating the transmission cables linear cost function.

The procedure is performed for each OSS candidate location. Thus, β_t linear functions are obtained, each characterized by a independent c_l^o , and non-negative linear c_l^v term, where $l \in \mathcal{N}_{\beta_t}$.

B. Objective function

After solving the collection and transmission systems sub-problems, the linear objective function of the main mathematical model is formulated as:

$$\min \sum_{i \in \mathcal{N} \setminus \{0\}} \sum_{j \in \mathcal{N}_w} \sum_{k=1}^{f(i)} c_{ij}^k \cdot y_{ij}^k + \sum_{i \in \mathcal{N}_\beta} \sum_{l \in \mathcal{N}_{\beta_i}} (c_l^o \cdot z_{il} + c_l^v \cdot \sigma_{il}) \quad (9)$$

C. Constraints

For a given set of candidate locations \mathcal{N}_{o_i} for the OSS i , exactly one of them must be chosen. This is modelled through

$$\sum_{l \in \mathcal{N}_{o_i}} z_{il} = 1 \quad \forall i \in \mathcal{N}_\beta \quad (10)$$

Note that in constraint (10), if an inequality (less than or equal to) replaces the equality, not only multiple locations would be supported, but also multiple number of OSSs.

In order to cluster the WTs (α) into β OSSs, the following constraint is required:

$$\sum_{i \in \mathcal{N}_\beta} \sigma_i = \alpha \quad (11)$$

In (11), σ_i represents the number of WTs associated to the OSS i , its definition is:

$$\sum_{l \in \mathcal{N}_{o_i}} \sigma_{il} = \sigma_i \quad \forall i \in \mathcal{N}_\beta \quad (12)$$

Correspondingly, σ_{il} counts the number of WTs connected to the OSS i at location l and is computed through

$$\sum_{j \in \mathcal{N}_w} \sum_{k=1}^{f(j)} k \cdot y_{lj}^k = \sigma_{il} \quad \forall i \in \mathcal{N}_\beta \wedge l \in \mathcal{N}_{o_i} \quad (13)$$

Variables σ_{il} are for linearization, while σ_i is for the model readability. The following equation, for both selecting an OSS location z_{il} and limiting the number of feeders out from them to ϕ , is added:

$$\sum_{j \in \mathcal{N}_w} \sum_{k=1}^{f(j)} y_{lj}^k \leq \phi \cdot z_{il} \quad \forall i \in \mathcal{N}_\beta \wedge l \in \mathcal{N}_{o_i} \quad (14)$$

The tree topology, i.e. only one cable type used per arc, and the definition of the head-tail convention, are simultaneously ensured by:

$$\sum_{i \in \mathcal{N} \setminus \{0\}} \sum_{k=1}^{f(i)} y_{ij}^k = 1 \quad \forall j \in \mathcal{N}_w \quad (15)$$

The flow conservation, which also avoids the formation of cycles (loops), is considered by means of one linear equality per wind turbine

$$\sum_{i \in \mathcal{N} \setminus \{0\}} \sum_{k=1}^{f(i)} k \cdot y_{ij}^k - \sum_{i \in \mathcal{N}_w} \sum_{k=1}^{f(i)} k \cdot y_{ji}^k = 1 \quad \forall j \in \mathcal{N}_w \quad (16)$$

The set \mathcal{I} stores pairs of arcs $\{(i, j), (u, v)\}$, which are crossing each other. Excluding crossing arcs in the solution is ensured by the linear inequalities

$$x_{ij} + x_{ji} + x_{uv} + x_{vu} \leq 1 \quad (17)$$

$$\forall \{(i, j), (u, v)\} \in \mathcal{I} : \{i, j, u, v\} \neq \emptyset$$

$$\sum_{k=1}^{f(i)} y_{ij}^k - x_{ij} \leq 0 \quad \forall (i, j) \in \mathcal{A}_r : \{i, j\} \neq \emptyset \quad (18)$$

Constraint (17) also includes the inverse arcs of those elements. This constraint is a practical restriction in order to

avoid hot-spots and potential single-points of failure caused by overlapped cables [21].

The following constraints represent a set of valid inequalities to tighten up the mathematical model [22]:

$$- \sum_{i \in \mathcal{N} \setminus \{0\}} \sum_{k=v+1}^{f(i)} \left\lfloor \frac{k-1}{v} \right\rfloor \cdot y_{ij}^k + \sum_{i \in \mathcal{N}_w} \sum_{k=v}^{f(i)} y_{ji}^k \leq 0 \quad (19)$$

$$\forall v \in \{2, \dots, U-1\} \wedge j \in \mathcal{N}_w$$

They can be interpreted as: given an active arc y_{ij}^k , the maximum number of active arcs rooted in j and connecting v WTs is expressed by $\left\lfloor \frac{k-1}{v} \right\rfloor$, hence the constraint restricts the maximum number of feasible arcs, reducing the search space without excluding valid solutions to the problem.

$$x_{ij} \in \{0, 1\} \quad y_{ij}^k \in \{0, 1\} \quad (20)$$

$$\forall (i, j) \in \mathcal{A}_r : \{i, j\} \neq \emptyset \wedge k \in \{1, \dots, f(i)\}$$

$$z_{il} \in \{0, 1\} \quad 0 \leq \sigma_i \leq \eta \cdot \left\lfloor \frac{\alpha}{\beta} \right\rfloor \quad \sigma_i \wedge \sigma_{il} \in \mathbb{Z}_+ \quad (21)$$

$$\forall i \in \mathcal{N}_\beta \wedge l \in \mathcal{N}_{o_i}$$

Constraints (20) and (21) define the nature of the formulation by the variables definition, a MILP. Note that variables σ_i are limited in their upper bounds to avoid overloading of OSSs (in case $\eta = 1$, if overloading is permitted then $1 < \eta \leq \beta$), which implicitly also bounds σ_{il} .

To summarize, the complete formulation of the main MILP model consists of the objective function (9) and constraints defined in (10) - (21).

III. MODELLING ASPECTS

The modelling choices presented in the following - with the exception of Section III-D - are based on the ones presented in [19] (particularly Section III-A, Section III-B, and Section III-C).

A. Cable capacity

The current capacity I_t of a cable t is calculated using the model given in [23]. This method comes from a Single-Core Equivalent Thermal Model (SCETM), as generalized in [24] for single-core and three-core cables. It is conservative when applied for OWFs, hence lifetime of the component is within acceptable intervals [25].

$$u_{c_t} = \left\lfloor \frac{S_{r_t} = \sqrt{3} \cdot V_{n_c} \cdot I_t}{P_n} \right\rfloor \quad \forall t \in \mathcal{T}_c \quad (22)$$

$$\mathcal{U}_c = \{1, \dots, u_{c_{|\mathcal{T}_c|}}\} \quad (23)$$

The set of cable capacities in terms of number of supportable WTs is defined in (22) and (23), where P_n represents the nominal power of an individual WT, and V_{n_c} the nominal line-to-line voltage level in the collection system. Regarding the transmission systems, the rated power of cables ($S_{r_t} \forall t \in \mathcal{T}_e$) is calculated with the voltage level V_{n_e} .

B. Arcs nominal power

A distributed model is implemented with the next expressions [26]

$$I_{ij}^{\vec{k},t} = \frac{k \cdot P_n}{\sqrt{3} \cdot V_{n_c}} \cdot \cosh(\vec{\gamma} \cdot d_{ij}) - \frac{V_{n_c}^{\vec{}}}{Z_c} \cdot \sinh(\vec{\gamma} \cdot d_{ij}) \quad (24)$$

$$S_{ij}^{k,t} = \sqrt{3} \cdot V_{n_c} \cdot |I_{ij}^{\vec{k},t}| \quad (25)$$

The characteristic impedance is calculated as $Z_c = \sqrt{\vec{z}/\vec{y}}$, and the propagation constant, $\vec{\gamma} = \sqrt{\vec{z} \cdot \vec{y}}$. The series impedance is represented by \vec{z} , and the admittance by \vec{y} . The maximum power flowing through the arc (i, j) , given k turbines connected downstream (y_{ij}^k), is calculated as per (25), accounting for the worst-case scenario, as the current in the arc is strictly increasing with length, and the value $I_{ij}^{\vec{k},t}$ is calculated at the extreme of it when using cable t .

For the transmission cables, (24) and (25) must be calculated per cable ($\frac{k \cdot P_n}{n}$), and considering V_{n_c} , to obtain $S_{0j}^{k,t,n}$.

C. Power flow and total power losses

A transportation model, accurate enough for radial systems, is implemented through Constraint (16). Time variation can be represented as:

$$\sum_{i \in \mathcal{N}} \sum_{k=1}^{f(i)} k \cdot y_{ij}^k \cdot p^\omega - \sum_{i \in \mathcal{N}_w} \sum_{k=1}^{f(i)} k \cdot y_{ji}^k \cdot p^\omega = 1 \quad (26)$$

$$\forall j \in \mathcal{N}_w \wedge \omega \in \Omega_\mu \wedge \mu \in \mathcal{M}$$

Constraint (16) is generalized considering the temporal dimension (26), where $\mathcal{M} = \{1, \dots, |\mathcal{M}|\}$, is the set of operational years with upper limit in the project lifetime, Ω_μ is the set of hours-slot for a year μ , and ω a specific hour-slot in Ω_μ . By means of the simulation of offshore power time series [27], let p^ω be the power in MW produced by one WT in that hour-slot.

In this way, let define the auxiliary variable $f_{ij}^{\omega,k} = k \cdot y_{ij}^k \cdot p^\omega$ as the power flow (MW) in arc (i, j) when k WTs are connected downstream (including the one in j), in time instant ω .

$$I_{ij}^{w,\vec{k},t} = \frac{f_{ij}^{\omega,k}}{\sqrt{3} \cdot V_{n_c}} \cdot \cosh(\vec{\gamma} \cdot d_{ij}) - \frac{V_{n_c}^{\vec{}}}{Z_c} \cdot \sinh(\vec{\gamma} \cdot d_{ij}) \quad (27)$$

$$l_{ij}^{\mu,t,k} \approx 3 \cdot (1 + \lambda_1 + \lambda_2) \cdot \sum_{\omega \in \Omega_\mu} w \cdot R_t \cdot d_{ij} \cdot |I_{ij}^{w,\vec{k},t}|^2 \quad (28)$$

$$+ 3 \cdot |\Omega_\mu| \cdot w \cdot W_{dt}$$

Including the capacitive currents, (27) expresses the current at the end of the arc (with respect to i), with magnitude $|I_{ij}^{w,\vec{k},t}|$. The annual total power losses $l_{ij}^{\mu,t,k}$ is calculated with (28). The factor $(1 + \lambda_1 + \lambda_2)$ accounts for the screen and armouring losses (with values from [23]), W_{dt} is the dielectric loss per unit length for the insulation surrounding the conductor in W/m (formula given in [23]), while the constant 3 is for the three-phase system. This value must be scaled in MWh.

Losses in transmission cables ($l_{0j}^{\mu,t,k,n}$) must consider the number of components in parallel per phase.

D. Forbidden areas

A restricted set of regions (inside a OWF) for excavation and cable trenching is supported in this model. Those regions can be modelled as convex polygons [28], non-convex polygons [29], or closed curves [30]. A convex hull based bypassing algorithm which may incorrectly delimit areas as forbidden, if they are non-convex polygons, is proposed in [28]. Any polygonal shape can be defined using Steiner nodes explicitly in the model as in [29], where the aim is not only to model the area, but also to refine arcs with shortest path; however, cables' bending moment may be compromised as result of unrealistic routing. Finally, more accurate computational algorithms to represent more precisely shapes is implemented in [30], by means of Delaunay Triangulation Based Navigational Mesh Path-finding; however, unrealistic routings can be resulted as well.

A pre-processing strategy in Fig. 2 is proposed in this manuscript, where any polygonal shape is modelled, while simultaneously decreasing the number of variables. The

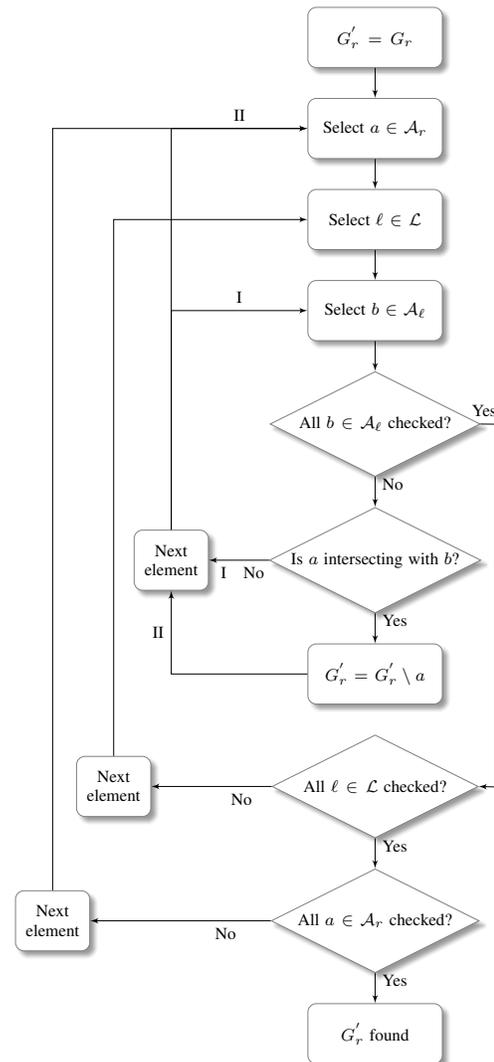


Fig. 2: Flowchart for incorporating forbidden areas in the model.

strategy is graphically represented, and avoids the explicit

creation of variables and constraints in the main model to cope with forbidden areas, and allowing only straight arcs between WTs. Let the set of forbidden areas be \mathcal{L} . An area $\ell \in \mathcal{L}$, in turn, is defined as a set of arcs \mathcal{A}_ℓ , where $b \in \mathcal{A}_\ell : b = (i, j)$. The set of arcs \mathcal{A}_ℓ defines a sequence of vertices enveloping the points representing a forbidden area ℓ (sequence $\{i, j, v, i\}$ represents, for example, a triangle with $\mathcal{A}_\ell = \{(i, j), (j, v), (v, i)\}$). The procedure for incorporating these zones into the model is depicted in Fig. 2, where graph G'_r is obtained from graph G_r , by excluding from the new arcs set \mathcal{A}'_∇ all the arcs crossing with at least one arc $b \in \mathcal{A}_\ell$.

IV. OPTIMIZATION FRAMEWORK

For large OWFs, i.e. in the range of 100s of WTs, the cable layout problem is generally unsolvable for gaps lower than 1% [31], therefore the graph G'_r needs further reduction to make the model tractable.

The framework proposed in [19] is implemented in this work. Let the function $f(i, G'_r, v)$ obtain the set Υ_i defined as maximum the v -closest WTs to i ; the term *maximum* accounts for vertices which have less than v arcs available due to prior elimination for crossings with forbidden areas. In other words, it is considered intuitively that a WT will be connected to one of the WTs in its vicinity. Therefore, by systematically applying $f(i, G'_r, v)$ to each $i \in \mathcal{N}_w$, the reduced graph G''_r is found. The set \mathcal{A}''_∇ contains the candidate arcs to the solution of the problem. The outcoming arcs from OSSs are also limited to the nearest 50 WTs. An imposed gap of ϵ is an input of the method.

The flowchart is displayed in the Fig. 3; as a first step a feasibility problem is solved, characterized by an objective function equal to zero, a low v to speed up the process, and fixed locations of OSSs by choosing arbitrarily one per each i available in \mathcal{N}_{oi} . Following the addressing of the feasibility

model given G''_r for a specific value of v . The next step is to compare if the best feasible point found so far, is included in the domain of the previously defined problem (\mathcal{A}''_∇); in this case, the process is stopped, since most likely the global minimum has been calculated; otherwise, the domain is further increased until that condition is satisfied. In an earlier work [19], it has been found out that for the feasibility problem $v = 5$, and for the main problem an initial value of $v = 15$, and increasing steps equal to 5, represent good parameters to find a proper compromise between computing time, and solution quality.

V. CASE STUDY

The following case studies have been carried out on an Intel Core i7-6600U CPU running at 2.50 GHz and with 16 GB of RAM. The chosen MILP solver is the branch-and-cut solver implemented in IBM ILOG CPLEX Optimization Studio V12.7.1 [32].

To test the validity of the model, a synthetic OWF (*SY*) with three forbidden areas is used. To test the ability of the model to solve very large instances, the real project London Array (*LA*), the second largest (measured by installed power) project under operation [33], is used. Both cases have more than 100 WTs and several OSSs, representing very challenges instances to solve.

The main high-level parameters for both case studies are given in Table I. The objective function is defined by a combined total economic cost including the initial investment, and the total electrical power losses of the cables systems (see (1) and (5)). Three forbidden areas are considered in *SY*, with four candidate locations per OSS, and distance to OCP of roughly 100 km, whereas in *LA* three possible locations are supported for each OSS, with length to OCP close to 20 km. In all cases, a maximum number of 10 feeders is allowed per OSS (see (14)), in addition to a balanced allocation of WTs (overloading of OSSs is accounted by means of $\eta = 1$, see (21)).

TABLE I
MAIN HIGH-LEVEL PARAMETERS FOR THE PROBLEM INSTANCES

OWF	Objective	r [%]	c_p [€/MWh]	$ \mathcal{M} $	$ \mathcal{L} $	η
<i>SY</i>	IL	5	40	30	3	1
<i>LA</i>						
OWF	P_n [MW]	\mathcal{N}_o	α	β	ϕ	
<i>SY</i>	3.6	$\{\{1, 2, 3, 4\}, \{5, 6, 7, 8\}\}$	120	2	10	
<i>LA</i>						$\{\{1, 2, 3\}, \{4, 5, 6\}\}$

The technical and economic parameters for both medium and high voltage cables are presented in Table II. Further cable's technical information available in [34]. For the collection systems, the set of cables is 240 mm², 500 mm², and 1,000 mm², while for transmission, ten different cables types are considered, ranging from 300 mm² to 2,000 mm². All the cross-sections are commercially standards by manufacturers. Cables capacities and costs are displayed for reproducibility as well.

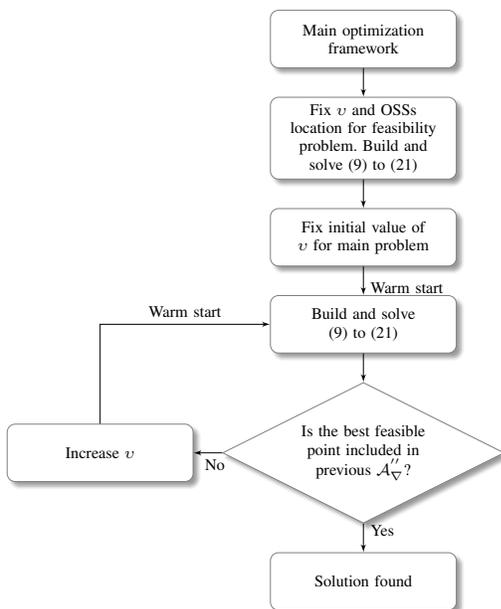


Fig. 3: Flowchart of the main optimization framework.

problem, the obtained solution is used to warm start the main

TABLE II
CABLES MAIN PARAMETERS

Collection system				
OWF V_{n_c} [kV]	$ \mathcal{T}_c $	\mathcal{U}_c	\mathcal{C}_c [M€/km]	
<i>SY</i>	33	3	{7, 10, 13}	{.36, .58, .90}
<i>LA</i>				
Transmission system				
OWF V_{n_e} [kV]	$ \mathcal{T}_e $	\mathcal{U}_e	\mathcal{C}_e [M€/km]	
<i>SY</i>	132	10	{34, 38, 42, 46, 49 53, 56, 59, 62, 69}	{.50, .56, .63, .73, .84 .96, 1.11, 1.28, 1.51, 2.16}
<i>LA</i>	150		{38, 43, 47, 52, 56 60, 63, 67, 71, 79}	{.57, .66, .78, .93, 1.13 1.33, 1.59, 1.91, 2.34, 3.60}

The linear cost function obtained using Fig. 1 for the first candidate location of OSS 1 in *SY* OWF is presented in Fig. 4 (distance to shore of 101.31 km). In this graph two set of curves are displayed. The first one (blue circles and line) includes the optimization objective function output when (5) - (8) are solved neglecting total electrical power losses (economic losses' discounted cash flow) in (5). The blue circles are the optimum cost of transmission cables from (5), while the blue line is the best linear curve fitting that data. In contrast, the second set of curves is determined including the total electrical power, where the red circles are from optimization objective function, and the red line is best linear data fitting.

Three main outcomes can be extracted from Fig. 4. (i) total

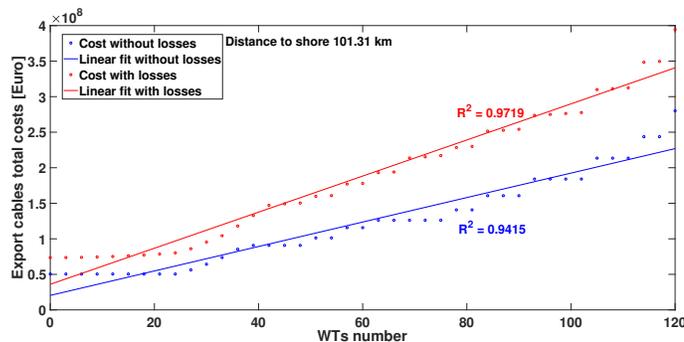


Fig. 4: Linear cost function with and without total electrical power losses.

electrical power losses boost considerably the total costs, as for instance, for 55 WTs an increase of 50% in costs is estimated. (ii) including electrical losses improves the linear function fit, as the R^2 value of the linear data fitting calculated with losses is higher than that without ($0.9719 > 0.9415$). This is because without taking electric losses into account, a larger nr. of WTs can be connected to the same cable size. The export route length for different OSSs candidate locations affects mainly the slope of the corresponding linear function, thus, the longer the length the higher the slope. Finally (iii), it can be seen for both sets that for low WTs numbers, i.e. between 1 and 40, the transmission cables minimum cost is poorly represented by the linear functions; however, it improves visibly for around 60 WTs and higher, which indicates that it can be used for the OWFs target size considered in this manuscript.

Overall, it is possible to infer that by including economic costs of electrical losses, not only a more holistic approach is supported (losses impact the layout as shown in [4], [35]), but also a statistical improvement is achieved, which enhances the accuracy of the model.

The graphical result of applying the integrated global optimization model for electrical cables in *SY* OWF is presented in Fig. 5. Candidate locations are indicated with red numbers. All hard constraints are abided, along with the non crossing of forbidden areas. Only cables with 240 and 500 mm² are used in the collection system. For the OSS, the locations closest to the OCP, hence minimizing the transmission cable length, are chosen. Each OSS is being connected with two 500 mm² cables.

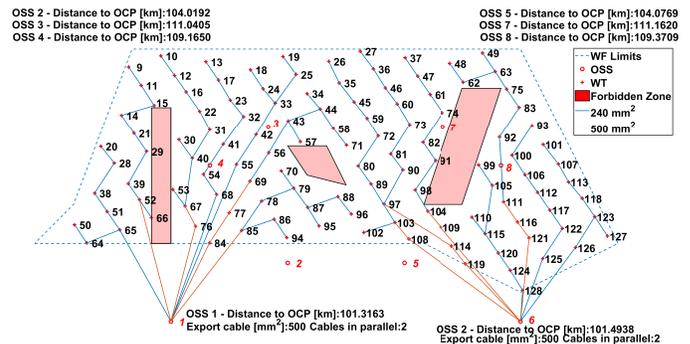


Fig. 5: *SY* OWF designed cable layout

Numerical results are available in Table III. For *SY* OWF ($\epsilon = 0.5\%$) a gap of 0.49% is achieved in 3 hours, and 31 minutes. An initial feasible solution is found in 77 seconds. In order to assess if the solver is able to take advantage of the problem's structure, an instance of *SY*, called as *SYs*, where only one candidate location is fixed per OSS (number 1 and 6 in Fig. 5), is solved by the model. A marginally equal solution is obtained in 1 hour, and 28 minutes. The total number of combinations in terms of OSSs pairs is 16, if one considers an average running time, then a total of 23 hours, and 31 minutes would be required. By means of the integrated model a reduction of 85% of running time is obtained.

TABLE III
MAIN RESULTS SUMMARY

Model	Best solution [M€]	Gap [%]	Total computation time [h]
<i>SY</i>	487.47	0.49	3.53
<i>SYs</i>	487.76	0.50	1.48
<i>LA</i>	144	0.75	23.29

Finally, the very large instance of *LA* OWF is tackled. The two previously described instances have been solved using the default settings offered by CPLEX. However, run-out-of-memory problems were faced when evaluating the *LA* instance. To cope with this limitation, a strong variables selection is used. This means that CPLEX invests considerable effort in analyzing potential branches of the nodes tree in the hope of drastically reducing the number of nodes that are explored. The

strategy compromises the running time but generally allows for less memory requirements. The LA OWF is solved with a gap of 0.75% in slightly more than 23 hours. In this case, the optimal solution results in OSSs located around the center of the OWF. This is intuitively valid, as the shorter length to shore (around 20 km) moves the weight towards minimizing the total length of the collector system cables.

VI. CONCLUSION

The proposed program provides an integrated global optimization approach to solve electrical cable systems of OWFs, particularly collection, and transmission systems. The contributions of this manuscript are manifold: (i) capability to harmonize and design both systems in a MILP model, (ii) fast computing time, and (ii) integration of realistic and high-fidelity physical models. The methodology has been validated against large-scale instances of OWFs projects. Several numerical results prove the validity and accuracy of the model in terms of abiding hard constraints within reasonable computation time, considering the complexity of the problems. A synthetic OWF (SY), and the second largest project in operation today, the London Array (LA), considering three candidate locations per OSS, have been used as case studies. The SY OWF is solved in around 3.5 hours with a global gap of 0.49%. The integrated model is roughly 85% faster than an enumeration approach. In the case of LA OWF, a strong variables selection is implemented, achieving a global gap of 0.75% in slightly more than 23 hours. The model indicates that the physical optimal location of OSSs is strongly affected by the distance to shore, as the export cables total costs starts becoming predominant for larger distances.

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